Combating Shadowing Effects for Systems with Transmitter Diversity by Using Collaboration among Mobile Users

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Abstract
In the current wireless, cellular systems, each mobile station (MS) independently communicates with a base station (BS). In this article we investigate collaboration among MSs where an MS retransmits a signal from the BS to a nearby MS that is in a shadowing or deep fading region relative to the BS. We develop a formula for the outage probability of a non-collaborative system with two transmit antennas and one receive antenna in a lognormal shadowing environment, and compare it with a formula developed for the outage probability of the collaborative system, in which the second transmit antenna is indeed located on another MS. Results of simulating a real system with practical parameters are also presented and compared with the theoretical results. The outage probability of the proposed system is reduced by an order of magnitude over that of the non-collaborative system.

I. Introduction
In a regular cellular system each MS communicates with the BS, without any knowledge of the other MSs’ situations. In this paper we investigate collaboration among MSs, in which adjacent MSs are able to assist each other under the supervision of the BS. Suppose an MS is situated in a deep fading region and receives a low-quality signal from the BS. In this case, the BS may employ a nearby MS to relay the signal to the weak MS (WMS). The intermediate MS is called a Collaborative MS (CMS). Figure 1 clarifies the idea of collaboration in which a CMS supports a WMS, which has an obstruction in its path to the BS.

An MS should satisfy two conditions to be a CMS:
1. It must be located inside a specific neighborhood of the WMS.
2. The received power by it should be above a specific threshold.

At first glance, it appears that the only change in the total power of the collaborative system, as compared to the original system, is the additional power consumed by the CMSs. But it can be shown that in a power-controlled situation, the overall power consumed in the system in fact decreases.

To avoid the coupling effect of the receive and the transmit antennas, it is necessary to separate the receiving and the transmitting channels in the CMS. This is possible by assigning a different frequency band to the retransmitted signal (in FDMA), a different time-slot (in TDMA), or a different code (in CDMA). The additional overhead in this case is expected to be a small fraction of the overall channel resources.

II. Overview and the System Parameters
To study the performance of the collaborative system, we have carried out several Monte-Carlo simulations. We consider a cellular wireless system in which MSs are uniformly distributed over the area of the cell (Figure 2).

The BS is located at the origin (0,0) and the cell area is
approximated with a circle of diameter $D$. The model for path loss is $\mu(r) = \mu_k - \beta \ln\left(\frac{r}{R}\right)$, and the shadowing model is lognormal. The parameters used in this paper are listed in Table 1.

Table 1. Description and values of the parameters used in formulas and simulations

<table>
<thead>
<tr>
<th>Para.</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D$</td>
<td>Cell Size (Diameter)</td>
<td>5000m</td>
</tr>
<tr>
<td>$d_n$</td>
<td>Neighborhood distance</td>
<td>250m</td>
</tr>
<tr>
<td>$N$</td>
<td>Total number of MSs</td>
<td>1000</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Shadow standard deviation</td>
<td>12dB</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Path loss exponent</td>
<td>5</td>
</tr>
<tr>
<td>$\mu_k$</td>
<td>Area mean power at edge of cell</td>
<td>-70dB</td>
</tr>
<tr>
<td>$\Omega$</td>
<td>Local mean power</td>
<td></td>
</tr>
<tr>
<td>$R$</td>
<td>Area mean power at distance $r$ from BS</td>
<td></td>
</tr>
<tr>
<td>$N_n$</td>
<td>Number of WMS’s neighborhood</td>
<td></td>
</tr>
</tbody>
</table>

### III. Probability of Finding a CMS

First, we calculate the probability of finding at least one nearby MS. For simplicity, in this section, the cell is approximated by a square of size $D$. Two MSs are nearby if the distance between them is less than a neighborhood distance $d_n$. Suppose there are $N$ MSs at the locations $(X_k, Y_k), k = 1, \ldots, N$, in which $X_k, Y_k$ are uniform random variables, i.e.

$$f_{X_k}(x_k) = U(-D/2, D/2), \quad f_{Y_k}(y_k) = U(-D/2, D/2),$$

where the probability density function (PDF) is denoted by $f$ and $U$ for uniform PDF. Two randomly selected MSs, assumed to be in locations $(x_1, y_1), (x_2, y_2)$, are in the same neighborhood if

$$(x_1 - x_2)^2 + (y_1 - y_2)^2 < d_n^2.$$  (1)

Given that $x_1, x_2, y_1, y_2$ are uniformly i.i.d. random variables, it can be shown that

$$p = \Pr\{(x_1 - x_2)^2 + (y_1 - y_2)^2 < d_n^2\},$$

will be given by

$$p = \frac{1}{2} \left(\frac{d_n}{D}\right)^4 - \frac{8}{3} \left(\frac{d_n}{D}\right)^3 + \pi \left(\frac{d_n}{D}\right)^2.$$  (2)

If there are $N$ MSs in the cell and $X$ is the number of the MSs in the same neighborhood of a randomly selected MS, then the PDF of $X$ is given by:

$$P_X(k) = \Pr(X = k) = \binom{N-1}{k} p^k (1-p)^{N-1-k}.$$  (3)

The probability of finding at least one nearby user is:

$$P_f = \sum_{k=1}^{N-1} P_X(k) = \sum_{k=1}^{N-1} \binom{N-1}{k} p^k (1-p)^{N-1-k},$$

or $1 - \Pr[\text{no nearby user}].$ Therefore

$$P_f = 1 - \Pr[\text{no nearby user}].$$  (3)

Previously we mentioned that a condition for an MS being a CMS is that the power it receives should be above the outage power. Assuming $P_{out}$ is the average outage probability in the cell, the probability of finding at least one CMS is given by:

$$P_{CMS} = (1 - P_{out}) P_f = (1 - P_{out}) [1 - (1-p)^{N-1}]$$  (4)

in which $p$ is given by Eq. (2).

Figure 3 compares the result of simulation with the formula given in (4). An interesting observation is that the total number of MSs in the cell should be roughly more than 400 for the probability of finding a CMS to be greater than 80%. This yields that CMSs cannot be chosen only among online users, which are fewer than 100 or so in a typical cellular system. MSs in standby also need to be employed as CMSs.

![Figure 3. Probability of finding at least one collaborative MS, versus the number of MSs in the cell.](image)

### IV. Outage Probability of the non-collaborative system

Because, the proposed collaborative system essentially entails the addition of a second transmitter, it is interesting to compare its performance with a two transmitter diversity system. Here we develop a formula for the outage probability of a system with two transmit antennas and one receive antenna. The results can be easily extended to the case of $M_t$ transmit and $N_r$ receive antennas. Figure 4 depicts the system in which the channel between transmit antenna zero and the receive antenna is denoted by $h_0$ and that between transmit antenna one and the receive antenna is denoted by $h_1$, where:
\[ h_0 = \alpha_0 e^{j\theta}, \quad h_i = \alpha_i e^{j\theta} \quad (5) \]

We define: \( S_j = \alpha_j^2 \) and \( S_i = \alpha_i^2 \). In a Rayleigh fading channel subjected to lognormal shadowing, the conditional cumulative density function (CDF) of these random variables is

\[ F_{S0}(s_i \mid \Omega) = 1 - e^{-\frac{s_i}{\bar{\nu}}}, \quad F_{S1}(s_i \mid \Omega) = 1 - e^{-\frac{s_i}{\bar{\nu}^2}}, \quad s_i > 0, \quad (6) \]

where \( \ln(\Omega) \) has a normal distribution of mean \( \mu(r) \) and shadowing variance \( \sigma^2 \). Therefore, the PDF of \( \Omega \) is

\[ f_\Omega(\Omega) = \frac{1}{\sqrt{2\pi \sigma \Omega^2}} e^{-\frac{(\ln(\Omega) - \mu(r))^2}{2\sigma^2}} \quad \Omega > 0. \quad (7) \]

\[ \text{Figure 4. Simple } 2 \times 1 \text{ transmit diversity system} \]

Here we assume that the probability of outage for an MS at distance \( r \) from the BS is the probability of both \( S_0 \) and \( S_i \) being below a specific threshold \( (S_{th}) \). For a given \( \Omega \) this probability is expressed as

\[ \text{Pr}(S_0 < S_{th} \text{ and } S_i < S_{th}) = \text{Pr}(S_0 < S_{th}) \cdot \text{Pr}(S_i < S_{th}) \]

\[ P_{out}(r \mid \Omega) = F_{S0}(S_{th} \mid \Omega) \cdot F_{S1}(S_{th} \mid \Omega) = (1 - e^{-\frac{S_{th}}{\bar{\nu}^2}})^2, \quad (8) \]

\[ P_{out}(r) \text{ is then given by} \]

\[ P_{out}(r) = \int_{-\infty}^{R} P_{out}(r \mid \Omega) f_\Omega(\Omega) d\Omega. \quad (9) \]

By substituting (8) and (7) in (9) we get

\[ P_{out}(r) = \int_{-\infty}^{R} (1 - e^{-\frac{S_{th}}{\bar{\nu}^2}})^2 \frac{1}{\sqrt{2\pi \sigma \Omega^2}} e^{-\frac{\ln(\Omega) - \mu(r)^2}{2\sigma^2}} d\Omega. \quad (10) \]

Finally, the average outage probability is

\[ P_{out} = \int_{-\infty}^{R} P_{out}(r \mid \Omega) f_\Omega(\Omega) d\Omega \]

in which \( R \) is the radius of the cell and \( f_\Omega(\Omega) \) is the PDF of \( \Omega \) assuming \( r \) is uniformly distributed over the cell area, we obtain

\[ P_{out} = \frac{1}{\pi R^2} \int_{-\infty}^{R} P_{out}(r \mid \Omega) 2\pi r dr = \frac{1}{R} \int_{-\infty}^{R} r P_{out}(r) dr. \quad (11) \]

Substituting (10) in (11) yields

\[ P_{out}(r) = \int_{-\infty}^{R} (1 - e^{-\frac{S_{th}}{\bar{\nu}^2}})^2 \frac{1}{\sqrt{2\pi \sigma \Omega^2}} e^{-\frac{\ln(\Omega) - \mu(r)^2}{2\sigma^2}} d\Omega. \quad (12) \]

The area mean power is obtained by \( \mu(r) = \mu_0 - \beta \ln(\frac{R}{r}) \), where \( \mu_0 \) is the area mean power at the edge of the cell and \( \beta \) is the path loss exponent. Further it can be shown that

\[ \frac{2}{R^2} \int_{0}^{R} r e^{-\frac{(\ln(\Omega) - \mu_0 - \beta \ln(\frac{R}{r}))^2}{2\sigma^2}} dr = \frac{1}{\beta} \int_{\Omega=0}^{1} \frac{2\sigma^2 + \beta \mu_0 - \beta \ln(\Omega)}{\sqrt{2} \beta \sigma} \Omega^{-\frac{\beta}{2}} \cdot Erfc(\frac{2\sigma^2 + \beta \mu_0 - \beta \ln(\Omega)}{\sqrt{2} \beta \sigma}) d\Omega \]

where \( Erfc \) is the complimentary error function. Using (12) and (13), the average outage probability is:

\[ P_{out} = \frac{1}{\beta} \int_{\Omega=0}^{1} \frac{1}{\Omega^{\frac{\beta}{2}}} (1 - e^{-\frac{S_{th}}{\bar{\nu}^2}})^2 \frac{2\sigma^2 + \beta \mu_0 - \beta \ln(\Omega)}{\sqrt{2} \beta \sigma} d\Omega \]

\[ (14) \]

V. Outage Probability of the Collaborative System

Figure 5 shows the baseband representation of the collaborative system. The channel between the transmit antenna and the collaborative MS \( k \) is \( h_k \), and the channel between the CMS and the receiver is \( h_{\text{rcv}} \), where

\[ h_{ki} = \alpha_{ki} e^{j\theta_k}, \quad h_{ij} = \alpha_{ij} e^{j\theta_i}, \quad k = 1, \ldots, N_s, \]

and \( N_s \) is the number of the nearby MSs to the weak MS. The CDF of \( \alpha_{ki}^2 \) and \( \alpha_{ij}^2 \) are given by:

\[ F_{aik}(s \mid \Omega_k) = 1 - e^{-\frac{s}{\bar{\nu}_k}}, \quad i = 1, 2. \]

Defining \( S_k = \alpha_{ki}^2 \alpha_{ij}^2 \) (a product of two exponential R.V.s), the CDF of the \( S_k \) is given by

\[ F_{S_k}(s \mid \Omega_k) = 1 - 2\sqrt{\frac{\Omega_k}{\Omega_{\text{th}}}} K_0(2\sqrt{\frac{\Omega_k}{\Omega_{\text{th}}}}) \quad s > 0, \quad \Omega_k > 0, \]

where \( \Omega = \sqrt{\Omega_1 \Omega_2} \) is a lognormal random variable and \( K_i(\cdot) \) is a modified Bessel function of the second kind. Since \( \Omega = \sqrt{\Omega_1 \Omega_2} \), then \( \ln(\Omega) = \frac{1}{2} (\ln(\Omega_1) + \ln(\Omega_2)) \).

Hence \( \ln(\Omega) \) is a normal random variable with

\[ \mu(r) = \frac{\mu_1 + \mu_2}{2}, \quad \sigma = \frac{\sigma_1 + \sigma_2}{2}. \]

Considering that the BS selects the best channel among \( N_s \) channels, we can write

\[ S' = \text{Max}_{k} S_k \quad k = 1, \ldots, N_s. \]

The CDF of the \( S' \) is then given by

\[ F_{S'}(s \mid \Omega, N_s) = \text{Pr}(S' < s) = \prod_{k=1}^{N_s} \text{Pr}(S_k < s) = \left[ F_{S_k}(s \mid \Omega_k) \right]^{N_s}, \]

therefore \( F_{S'}(s \mid \Omega, N_s) = \left[ 1 - 2\sqrt{\frac{\Omega_k}{\Omega_{\text{th}}}} K_0(2\sqrt{\frac{\Omega_k}{\Omega_{\text{th}}}}) \right]^{N_s}. \)
where \( N_s = 1, \ldots , N \), in which \( N \) is the total number of the users in the cell. \( N_s \) has a binomial distribution. Averaging over all possible values of \( N_s \) we get

\[
F_s(s | \bar{\Omega}) = \sum_{N_s=0}^{N-1} \left[1 - \frac{2\sqrt{s}}{\bar{\Omega}} K_0\left(\frac{2\sqrt{s}}{\bar{\Omega}}\right)\right]^{N_s} \left(\frac{N-1}{N_s}\right) p^{N_s} (1-p)^{N-1-N_s},
\]

in which \( p \) is the probability of two randomly selected MSs being nearby and the value of the \( p \) is given by the Eq. (2). Symplifying (15), we get:

\[
F_s(s | \bar{\Omega}) = \left[1 - p \frac{2\sqrt{s}}{\bar{\Omega}} K_0\left(\frac{2\sqrt{s}}{\bar{\Omega}}\right)\right]^{N-1}.
\]

Integrating (16) over all possible \( \bar{\Omega} \) yields

\[
F_s(s) = \int_{0}^{\infty} \left[1 - p \frac{2\sqrt{s}}{\bar{\Omega}} K_0\left(\frac{2\sqrt{s}}{\bar{\Omega}}\right)\right]^{N-1} \frac{1}{\sqrt{2\pi \sigma \Omega}} e^{-\frac{(\ln(\Omega) - \bar{\mu}(s))^2}{2\sigma^2}} d\Omega.
\]

Using the same procedure for channel zero (BS straight to WMS), the outage probability is obtained from

\[
P_{out}^0 = \frac{1}{\beta} e^{-\frac{2\sigma^2 + \bar{\beta} \bar{\mu}_s - \beta \ln(\Omega)}{\sqrt{2} \beta \sigma}} \text{erfc}\left(\frac{\beta \mu_s - \beta \ln(\Omega)}{\sqrt{2} \beta \sigma}\right),
\]

(19)

Since channels one and zero are independent, the overall average outage probability is the product of (18) and (19)

\[
P_{out} = P_{out}^0 P_{out}^1
\]

(20)

Figure 6 depicts the outage probability as modeled by the simulation as well as the theoretical calculation – Eqs. (14) and (20) – for collaborative and non-collaborative two-transmitter systems. The figure shows that the proposed system’s outage probability is roughly 10 times better for a reasonable threshold power, \(-80\text{dB}\).

VI. Conclusions

In this paper we proposed an approach of using mobile stations as relays between a base station and a mobile station that is in a deep shadow or fade situation. The performance of this system is compared, theoretically and by simulations, with that of system employing two transmit antennas under shadowing. The outage probability, an important criterion of system performance, of a collaborative system is roughly 10 times better than the non-collaborative system.

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Reference